AQA

AS

Mathematics

MFP1 – Further Pure 1

Mark scheme

6360

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$\alpha+\beta=6$; $\alpha\beta=14$	B1; B1	2	If LHS is missing look for later evidence before awarding the B1s
(b)	$P=\dfrac{\alpha}{\beta}\times\dfrac{\beta}{\alpha}=1$	B1		$P=1$ seen or used
	$S=\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=\dfrac{\alpha^2+\beta^2}{\alpha\beta}$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=36-28=8$	M1		$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ OE seen or used
	$S=\dfrac{8}{14}$	A1		A correct value for S seen, or used in quadratic. Ft on wrong sign for $\alpha+\beta$.
	x^2-Sx+P $(=0)$	M1		Using correct general form of LHS of eqn **with** ft substitution of c's S and P non-zero values.
	$x^2-\dfrac{8}{14}x+1$ $(=0)$ (Quadratic eqn is) $7x^2-4x+7=0$	A1	5	CSO. ACF of the equation, but must have integer coefficients
	Total		7	
(b)	A possible OE for 1st M1: $\alpha^2+\beta^2=6(\alpha+\beta)-14-14$			
Altn (b)	$Y=x^2/14$ **(B1)** A subst of this Y attempted in given quadratic **(M1)** $14Y+14=6x$ **;** $(14Y+14)^2=36(14Y)$ **(m1** full subst); A correct eqn no brackets, square roots or fractions **(A1);** as main scheme **(A1cso)**			

Q2	Solution	Mark	Total	Comment
(a)	$\{y(2+h)=\}$ $[2-(2+h)](1+2+h)+3$	M1		Attempt to find y when x=2+h. PI
	Gradient = $\dfrac{-h(3+h)+3-3}{2+h-2}$	M1		Use of gradient = $\dfrac{y_2-y_1}{x_2-x_1}$ OE to obtain an expression in terms of h.
	$=\dfrac{-h(3+h)}{h}=-3-h$	A1	3	CSO $-3-h$ or $-(3+h)$ or equally simplified.
(b)	As $h\to 0$, {grad. of line in **(a)** $\to$ grad. of curve at (2, 3)}	E1		'$h\to$ 0' seen OE in words NB 'h = 0' instead of '$h\to$ 0' gets E0
	{Gradient of curve at point (2, 3) =} -3	A1F	2	ft on c's a value only if both M1s have been scored in part **(a)** and a+bh has been obtained convincingly for non zero a and b Final answer left as '$\to -3$' is A0
	Total		5	
(b)	Differentiation to find dy/dx = -3 when x=2 scores E0A0F			
(b)	Note: E0, A1F is possible			
(b)	Marking the **E1**….OE wording for '$\to$' eg 'tends to', 'approaches', 'goes towards'. Do NOT accept '='			
(b)	Example: As $h\to$ 0 gradient $\to -3-0=-3$ (E1A0F)…if cand had then written 'gradient is/= -3' the A1F would have been scored			

Q3	Solution	Mark	Total	Comment
(a)	$\log_{10} y = \log_{10} a + \log_{10} b^x$ $\log_{10} y = \log_{10} a + x\log_{10} b$	M1		$\log ab^x = \log a + \log b^x$ seen or used
	$Y = \log_{10} a + x\log_{10} b$ (is a linear relationship between Y and x)	A1	2	$Y = \log_{10} a + x\log_{10} b$
(b)(i)	(gradient of line =) -0.4	B1	1	Correct value for gradient
(b)(ii)	$\log_{10} a = 2.5$, $a = 10^{2.5}$	M1		$\log_{(10)} a = 2.5$ OE PI by $a = 316$
	$a = 316$ (to 3 sf)	A1		CAO $a = 316$
	$\log_{10} b$ = gradient of line = -0.4	M1		$\log_{(10)} b = -0.4$ OE ft c's **(b)(i)** answer **OR** $b^5 = 10^{-2}$ OE PI by $b = 0.398$
	$b = 0.398$ to 3 sf	A1	4	CAO $b = 0.398$
	Total		**7**	
(a)	If base 10 is missing or if $\log_{10} y$ has not been replaced by Y then A0			

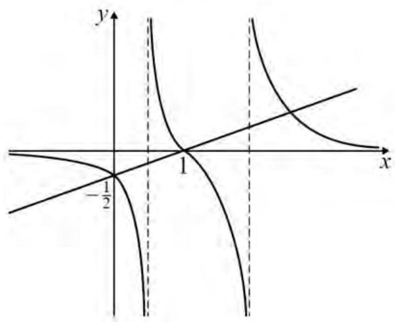
Q4	Solution	Mark	Total	Comment
(a)	$k=6$	B1	1	A correct value of k. Either 6 or -6
(b)	$\cos\left(2x-\frac{5\pi}{6}\right)=\cos\frac{\pi}{6}$	M1		$\cos\left(2x-\frac{5\pi}{6}\right)=\cos\frac{\pi}{k}$, stated or used; if incorrect ft c's k value. **Altn** $\sin\left[\frac{\pi}{2}-\left(2x-\frac{5\pi}{6}\right)\right]=\sin\frac{\pi}{3}$ (*) OE
	$2x-\frac{5\pi}{6}=2n\pi+\frac{\pi}{6}$, $2x-\frac{5\pi}{6}=2n\pi-\frac{\pi}{6}$	m1		OE Either one, showing a correct use of $2n\pi$ in forming a general solution. If incorrect ft c's k value. **Altn** using (*) above, $X=2n\pi+\frac{\pi}{3}$ or $X=2n\pi+\pi-\frac{\pi}{3}$ OE where $X=\frac{\pi}{2}-\left(2x-\frac{5\pi}{6}\right)$. Condone $360n$ for $2n\pi$ in both methods
	$x=\frac{1}{2}\left(2n\pi+\frac{5\pi}{6}\pm\frac{\pi}{6}\right)$	A1F		Full sets of GS, if incorrect ft c's k value, condoning unsimplified forms ie check $x=\frac{1}{2}\left(2n\pi+\frac{5\pi}{6}\pm\frac{\pi}{k}\right)$. (**A0F** if degrees present in answer)
	$x=n\pi+\frac{\pi}{2}$, $x=n\pi+\frac{\pi}{3}$	A1	4	OE full set of correct solutions in rads with constant terms combined. **If using the Altn, final 2 marks become A2,1,0**
(c)	$\tan x=\tan(n\pi+\frac{\pi}{2})=\tan\frac{\pi}{2}$ not finite $\tan x=\tan(n\pi+\frac{\pi}{3})=\tan\frac{\pi}{3}$ (ie a single finite value)	E1		Considers the complete set of general solutions from (**b**), showing that one results in non finite value for tan x and the other gives single value. Must be working with general n and must refer to either 'finite' or 'non finite'
	(Only possible finite value for tan x is) $\sqrt{3}$	B1	2	$\sqrt{3}$ This B1 mark is dep on k=6 and at least 3 marks scored in part **(b)** but **not** dependent on E1.
	Total		7	
(b)	Example: $\cos\left(2x-\frac{5\pi}{6}\right)=\cos\frac{\pi}{6}$, $2x-\frac{5\pi}{6}=\frac{\pi}{6}$, $2x=\pi$, $2x=2n\pi\pm\pi$, $x=n\pi\pm\frac{\pi}{2}$ (**M1m0**)			

Q5	Solution	Mark	Total	Comment
(a)	$\sum_{r=1}^{n}(6r-3)^2 == \sum_{r=1}^{n}\left(36r^2-36r+9\right)$			
	$= 36\sum_{r=1}^{n}r^2 - 36\sum_{r=1}^{n}r + 9\sum_{r=1}^{n}1$	M1		$\sum_{r=1}^{n}\left(\alpha r^2+\beta r\right)=\alpha\sum_{r=1}^{n}r^2+\beta\sum_{r=1}^{n}r$ seen/used
	$= 36\sum_{r=1}^{n}r^2 - 36\sum_{r=1}^{n}r + 9n$	B1		$\sum_{1}^{n}1=n$ seen or used
	$=36\left\{\frac{n}{6}(n+1)(2n+1)-\frac{n}{2}(n+1)\right\}+9n$	m1		Subst of correct expressions for $\sum_{1}^{n}r^2$ and $\sum_{1}^{n}r$
	$= 6n(n+1)[2n+1-3]+9n$			
	$= 3n[4(n+1)(n-1)+3]$	A1		OE Correct $3n[....]$ convincingly obtained before printed answer, where [….] would reduce to $4n^2-1$ or $12n^3-3n$ obtained convincingly
	$= 3n\left(4n^2-1\right)$	A1	5	AG. $3n\left(4n^2-1\right)$ convincingly obtained
(b)	$\sum_{r=1}^{2n}r^3=\frac{(2n)^2}{4}(2n+1)^2$	B1		OE correct expression for $\sum_{r=1}^{2n}r^3$ stated or used
	$\sum_{r=1}^{2n}r^3-\sum_{r=1}^{n}(6r-3)^2=$ $n^2(2n+1)^2-3n\left(4n^2-1\right)$			
	$= n(2n+1)[n(2n+1)-3(2n-1)]$	M1		Either $n(2n+1)\mathrm{g}(n)$, where g(n) is a quadratic OR reaching an equivalent stage in the factorisation of the correct quartic $4n^4-8n^3+n^2+3n$ in n.
	$= n(2n+1)(2n^2-5n+3)$	A1		OE A correct expression in n with product containing at least two linear factors with the quadratic factor simplified.
	$= n(2n+1)(2n-3)(n-1)$	A1	4	Correct product of four linear factors in n.
	Total		9	
(a)	$\sum_{r=1}^{n}9=9$ seen or used at any stage will result in B0A0A0			
(b)	For penultimate A1 OEs <u>include</u> eg $n(n-1)\left(4n^2-4n-3\right)$; $n(2n-3)\left(2n^2-n-1\right)$;			

Q6	Solution	Mark	Total	Comment
(a)	C: $(y-3)^2 = 4a(x-2)$	M1		$(y-3)^2 = 4a(x-2)$ OE PI by next line
	$(7-3)^2 = 4a(4-2)$	A1		OE
	$16 = 8a$, $a = 2$	A1	3	AG Be convinced
(b)	$(y-3)^2 = 4a(ky-2)$	M1		Replacing x by ky or y by $\frac{x}{k}$ in c's eqn for C.
	$y^2 - (6+4ak)y + 9 + 8a = 0$	A1		A correct quadratic eqn in the form either $Ay^2 + By + C = 0$ or $Ax^2 + Bx + C = 0$ PI by later work.
	$B^2 - 4AC = [-(6+4ak)]^2 - 4(9+8a)$	M1		$B^2 - 4AC$ in terms of k (condone a remaining); ft on c's quadratic provided relevant coefficient(s) are in terms of k and A, B, C are all non zero.
	Roots non-real $\Rightarrow B^2 - 4AC < 0 \Rightarrow$ $[-(6+8k)]^2 - 4(25) < 0$	A1		A correct strict inequality where k is the only unknown (a must be replaced by 2 by this stage)
	$(4k+8)(4k-2) < 0$; (*) critical values $k = -2$, $k = 0.5$	A1		Correct critical values stated or used and correctly obtained.
	((*) true for) $-2 < k < 0.5$, (the values for which line does not meet curve C.)	A1	6	$-2 < k < 0.5$
	Total		**9**	
ALTn (a)	(4,7) from translating $(4-2, 7-3)$ ie (2,4) on parabola $y^2 = 4ax$ **(M1)**; $4^2 = 4a(2)$ **(A1)**; a=2**(A1**above**)**			
(b)	Quadratic in x: eg $\frac{x^2}{k^2} - \left(\frac{6}{k} + 4a\right)x + 9 + 8a = 0$			
ALTn (b)	Translating the line backwards to link with $y^2 = 4ax$: ie working with $y^2 = 4ax$ and $k(y+3) = x+2$ gives eg $y^2 - 4aky - 12ak + 8a = 0$ then $2k^2 + 3k - 2 < 0$ etc			

Q7	Solution	Mark	Total	Comment
(a)	$(x+2)^2-4+20=0$	M1		OE eg $(x+2)^2=-16$
	$x+2=\pm 4\mathrm{i}$	B1		$\sqrt{-16}=4\mathrm{i}$
	$(x=)\quad -2\pm 4\mathrm{i}$	A1	3	$-2\pm 4\mathrm{i}$ NMS $-2\pm 4\mathrm{i}$ scores 3 marks.
Altn (a)	$(x=)\ \dfrac{-4\pm\sqrt{16-4(20)}}{2}\ \left\{=\dfrac{-4\pm\sqrt{-64}}{2}\right\}$	(M1)		Correct substitution into quadratic formula
	$=\dfrac{-4\pm 8\mathrm{i}}{2}$	(B1)		$\sqrt{-64}=8\mathrm{i}$ or $\dfrac{\sqrt{-64}}{2}=4\,\mathrm{i}$
	$(x=)\quad -2\pm 4\mathrm{i}$	(A1)	(3)	$-2\pm 4\mathrm{i}\quad (c=-2,\ \ d=\pm 4)$
(b)(i)	Roots are complex conjugates (and coeff. of z^2 and constant term are both real) so coefficients of quadratic are all real	E1		Altn: $w+w^*=2\,\mathrm{Re}(w)$ $w+w^*=(-b/a)=-(4+i+iq)$
	$(4+\mathrm{i}+q\,\mathrm{i})$ is real ie for real q $(1+q)\,\mathrm{i}=0\Rightarrow q$ must be -1.	E1	2	Indep of previous E1 but must refer to $(4+\mathrm{i}+q\,\mathrm{i})$ or coefficient of z being 'real' and $q=-1$
(b)(ii)	Roots $p+2\mathrm{i}$ and $p-2\mathrm{i}$,	B1		PI by subst of both $p+2\mathrm{i}$ and $p-2\mathrm{i}$ for z or $(p+2\mathrm{i})(p-2\mathrm{i})$ seen or $(p+2\mathrm{i})+(p-2\mathrm{i})$ seen
	$(p+2\mathrm{i})(p-2\mathrm{i})=20\Rightarrow\ p^2=16$	M1		Either or equivalent
	$(p+2\mathrm{i})+(p-2\mathrm{i})=-4-\mathrm{i}-q\,\mathrm{i}$ $\Rightarrow \pm 8=-4-\mathrm{i}-q\,\mathrm{i}$	M1		OE eg q must be in the form $-1+k\mathrm{i}$, where k is real. $\pm 8=-4+k$
	$q=-1+12\mathrm{i}$	A1		
	$q=-1-4\mathrm{i}$	A1	5	
	Total		10	
(a)	**Altn** $2c=-4$, $c^2+d^2=20$ (**M1** need both); $c=-2$ (**B1**) $d=\pm 4$; $-2\pm 4\mathrm{i}$ (**A1**)			

Q8	Solution	Mark	Total	Comment
(a) (i)	$(\mathbf{A}^2 =) \quad \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$	B1	1	
(a)(ii)	Stretch parallel to x-axis scale factor 4	B1	1	OE
(b)	$y = -\frac{1}{\sqrt{3}}x = \tan(-30^\circ)x$ $\mathbf{B} = \begin{bmatrix} \cos(-60^\circ) & \sin(-60^\circ) \\ \sin(-60^\circ) & -\cos(-60^\circ) \end{bmatrix}$	M1		Attempting to write $x + \sqrt{3}y = 0$ in the form $y = x\tan\theta$, PI then seeing/using $\mathbf{B} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$ with a value of θ such that $\tan\theta = -\frac{1}{\sqrt{3}}$ or $\frac{1}{\sqrt{3}}$
	$\mathbf{B} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$	A1	2	Any correct exact non-trig form
(c)	$\mathbf{BA}^2 = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$	M1		Setting up the product of c's $\mathbf{B}$ and c's $\mathbf{A}^2$ in any order.
	$\begin{bmatrix} 2 & -\frac{\sqrt{3}}{2} \\ -2\sqrt{3} & -\frac{1}{2} \end{bmatrix} \left(\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \end{bmatrix}\right)$	m1		Multiply c's $\mathbf{B}$ by c's $\mathbf{A}^2$ in correct order to obtain a 2 by 2 matrix. PI by correct 2 by 1 matrix for $\mathbf{BA}^2\begin{bmatrix} x \\ y \end{bmatrix}$ in terms of x and y.
	$2x - \frac{\sqrt{3}}{2}y = 0$	A1		OE
	$-2\sqrt{3}x - \frac{1}{2}y = -4$	A1		OE
		m1		Solving two correct equations to find a correct value for either the x-coordinate or the y-coordinate of P.
	$P\left(\frac{\sqrt{3}}{2}, 2\right)$	A1	6	Correct coordinates. Condone answer left as eg $x = \frac{\sqrt{3}}{2}$, $y = 2$ but do not accept answer left as a matrix.
	Total		10	
Altn (c)	$\mathbf{BA}^2\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$ **(M1)** PI; $\mathbf{B}^2\mathbf{A}^2\begin{bmatrix} x \\ y \end{bmatrix} = \mathbf{A}^2\begin{bmatrix} x \\ y \end{bmatrix}$ ($\mathbf{B}^2 = \mathbf{I}$ seen/used **(m1)**) $\mathbf{A}^2\begin{bmatrix} x \\ y \end{bmatrix} = \mathbf{B}\begin{bmatrix} 0 \\ -4 \end{bmatrix}$ (**m1** with both products attempted**) ;** $\begin{bmatrix} 4x \\ y \end{bmatrix} = \begin{bmatrix} 2\sqrt{3} \\ 2 \end{bmatrix}$ **(A1 LHS)** **(A1 RHS);** $P\left(\frac{\sqrt{3}}{2}, 2\right)$ **A1**			

Q9	Solution	Mark	Total	Comment
(a)	$x=2$; $x=0.5$; $y=0$	B2,1,0	2	OE . **B1** for two correct equations and no more than one incorrect equation.
(b)	$\frac{1}{2}(x-1)=\frac{x-1}{(x-2)(2x-1)}$	M1		Correct elimination of y
	$\frac{1}{2}(x-1)(x-2)(2x-1)=x-1$	A1		Any correct cubic
	$(x-1)(x-2)(2x-1)=2(x-1)$ $(x-1)[(x-2)(2x-1)-2]=0$ $(x-1)[2x^2-5x]=0$ $x=0$, $x=1$, $x=2.5$	A1	3	From a relevant factorised form or from $2x^3-7x^2+5x=0$ obtained from correct working
(c)		B1		C: 3-branch curve, no parabolas, no branch having positive slopes. Condone slight deviations at the two horizontal extremes.
		B1		C: correct curve with correct asymptotic behaviour with correct asymptotes seen/implied
		B1	3	L: correct 'line', intersecting a 3-branch curve at 3 points, two of which are on the coordinate axes.
(d)	$x\le 0$, $0.5<x\le 1$, $2<x\le 2.5$	M1		Three inequalities consistent with the c's 3-branch curve C and line L with positive slope drawn in part **(c)**, ft three values of x obtained in **(b)** used with correct values for vertical asymptotes, condoning $<$ for $\le$ and vice versa.
		A2,1	3	**A2** all three inequalities correct. **A1** if only error is either one or both '$<$' replaced by '$\le$' or one '$\le$' replaced by $<$
	Total		11	